Nuclear Effects in Neutrino DIS Calculations

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Outline

- Sketch of experimental evidence of nuclear effects in DIS.
- Brief overview of different sources of nuclear corrections to structure functions.
  - Incoherent scattering off bound nucleons (IA).
  - Modification of bound nucleon = off-shell effect.
  - Nuclear pion effect. Nuclear shadowing.
- DIS sum rules as important constraints in calculations.
- Results for nuclear neutrino structure functions.

Experimental evidence of nuclear effects in DIS

- Data on nuclear effects in DIS are available in the form of the ratio $\mathcal{R}_2(A/B) = F_2^A/F_2^B$.

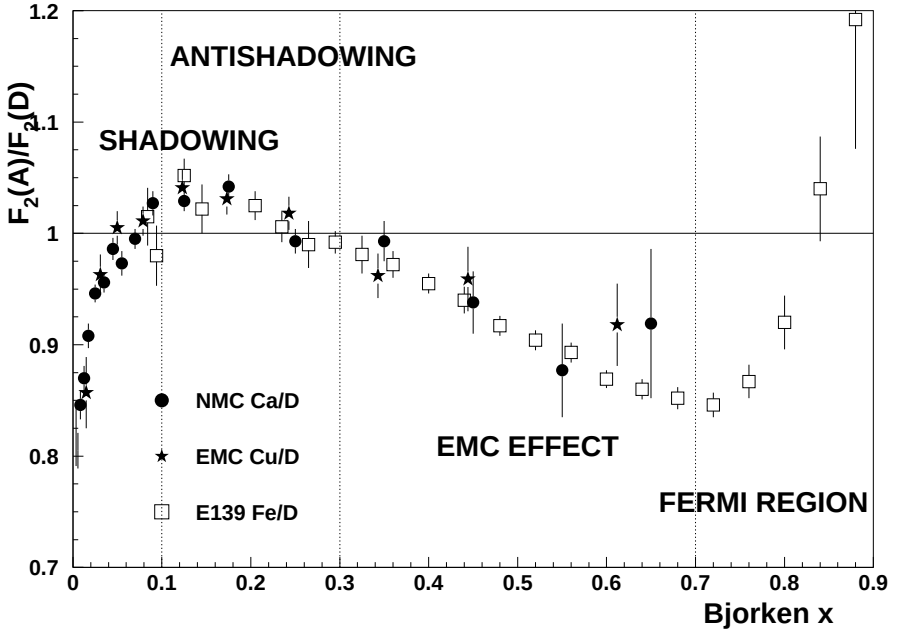
- Targets:
  Variaty of nuclear targets from the deuteron ^{2}D to lead ^{208}Pb

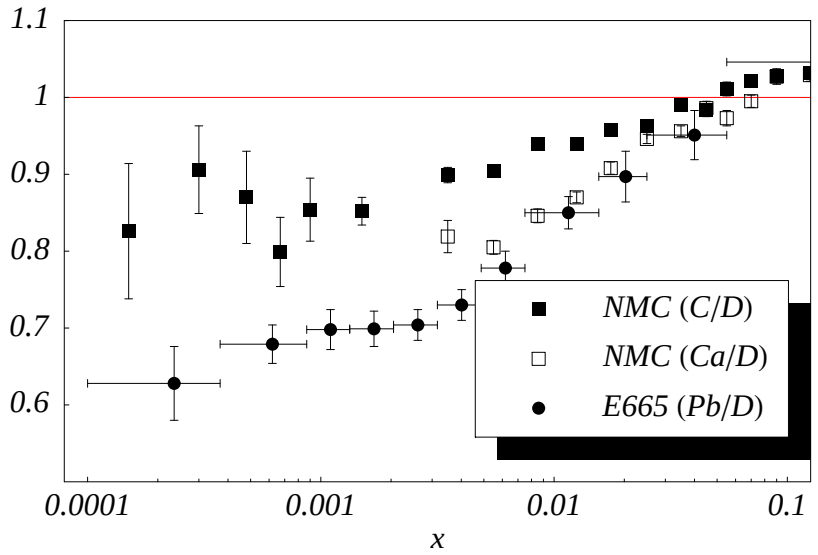
- Experiments:
  – Muon beam at CERN (EMC, BCDMS, NMC) and FNAL (E665).
  – Electron beam at SLAC (E139, E140), HERA (HERMES) and recently at JLab.

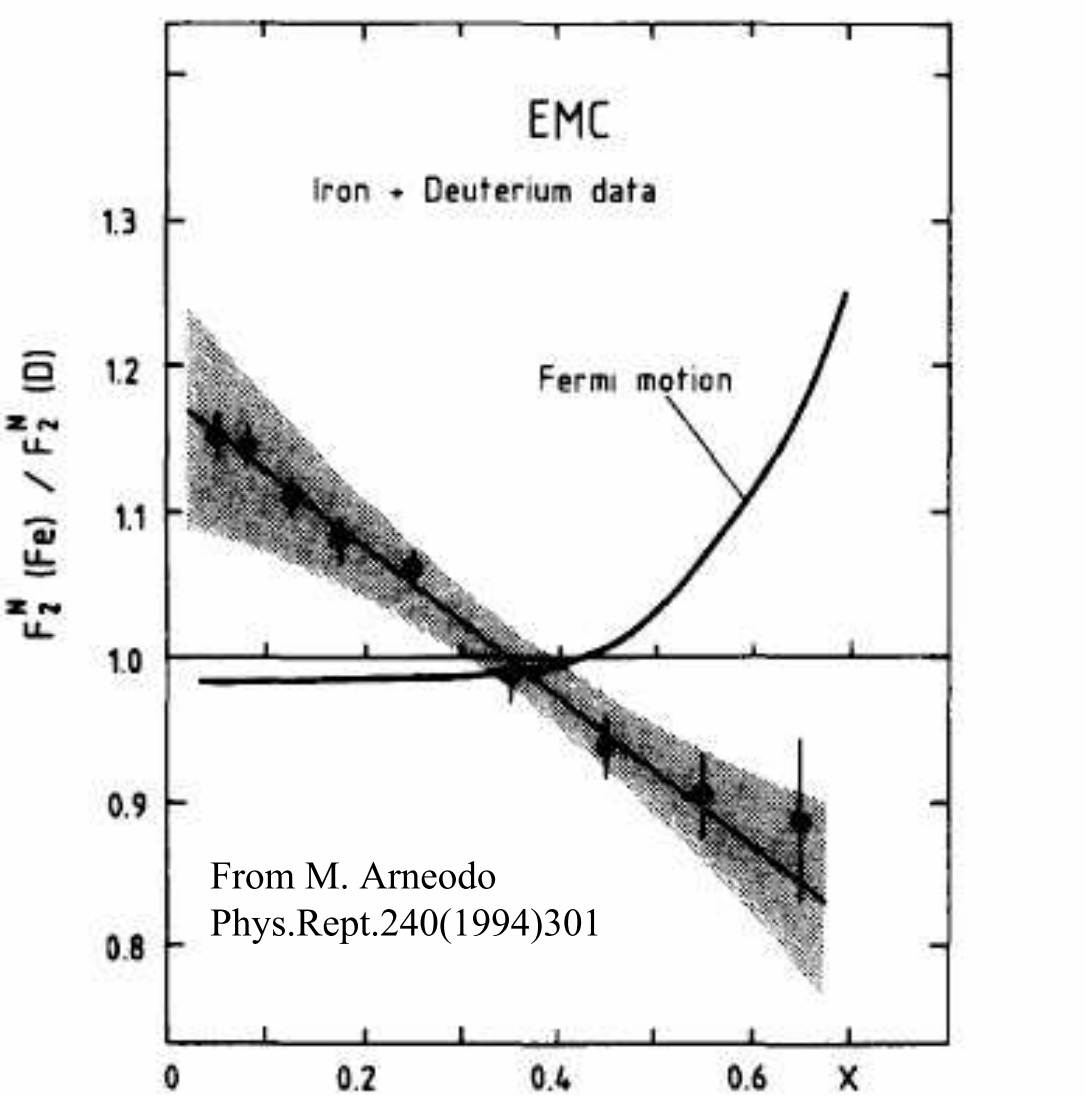
- Kinematics and statistics:
  Data covers the region $10^{-4} < x < 0.9$ and $0 < Q^2 < 150\,\mathrm{GeV}^2$. There available about 600 data points with $Q^2 > 1\ \mathrm{GeV}^2$.

Data on $\mathcal{R}$ show a weak Q^2 dependence that suggests scaling origin of (at least a part of) nuclear effects. Characteristic nuclear effects were observed for different kinematical regions of x.

- Nuclear shadowing at small values of Bjorken x ($x < 0.05$).
- Antishadowing at $0.1 < x < 0.25$.
- A well with a minimum at $x \sim 0.6 \div 0.75$ (EMC effect).
- Enhancement at large $x > 0.75 \div 0.8$ (Fermi motion region).






EMC
Iron + Deuterium data
Fermi motion
F_2^N (Fe) / F_2^N (D)
1.3
1.2
1.1
1.0
0.9
0.8
0
0.2
0.4
0.6
X
From M. Arneodo
Phys.Rept.240(1994)301

Why Nuclear Effects Survive in DIS ?

Typical regions in configuration space which contribute to DIS hadronic tensor:

- $t^2 - z^2 \sim Q^{-2}$ DIS proceeds near the light cone.
- $t \sim z \sim L = (Mx)^{-1}$ NOT small in hadronic scale (in the target rest frame)
  $\Rightarrow$ the reason for nuclear corrections to survive even at high Q^2.

L has to be compared with average distance between bound nucleons $d = (3/4\pi\rho)^{1/3} \sim 1.2\,\mathrm{Fm}$ (central region of heavy nuclei). One should distinguish two different regions:

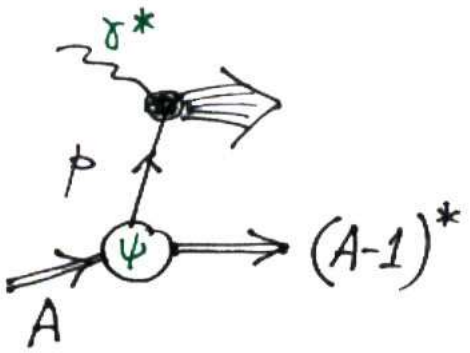
- $L < d$ $\Rightarrow$ Nuclear DIS $\approx$ incoherent sum of contributions from bound nucleons.
- $L \gg d$ $\Rightarrow$ Coherent effects of interactions with a few nucleons are important.

Incoherent Scattering from Bound Nucleons

Fermi motion and nuclear binding corrections (FMB)

$$F_2^A(x,Q^2) = \int \mathrm{d}^4k\, \mathcal{P}_A(k) \left(1+\frac{k_z}{M}\right) F_2^N(x',Q^2,k^2),$$

$$x = \frac{Q^2}{2p\cdot q}, \quad x' = \frac{Q^2}{2p\cdot q} = \frac{x}{1+(\varepsilon+k_z)/M}$$



Similar equations hold in impulse approx. for other structure functions (F_T, F_3). Fermi motion and binding effect is driven by nuclear spectral function

$$\mathcal{P}_A(k) = \sum_n |\psi_n(\boldsymbol{k})|^2 \delta(\varepsilon + E_n(A-1) - E_0(A)).$$

Spectral function describes probability to find a bound nucleon with momentum $\boldsymbol{k}$ and energy $k_0 = M+\varepsilon$.

Nuclear spectral function

Mean-field picture

Nucleus in a first approximation can be viewed as a system of protons and neutrons bound to a self-consistent potential (mean field model, MF). Nucleons occupy the MF energy levels according to Fermi statistics and thus distributed over momentum (Fermi motion) and energy states. MF nuclear spectral function

$$\mathcal{P}_{\rm MF}(\varepsilon,\boldsymbol{p}) = \sum_{\lambda<\lambda_F} n_\lambda |\phi_\lambda(\boldsymbol{p})|^2 \delta(\varepsilon-\varepsilon_\lambda)$$

where sum is taken over occupied levels with ϕ_λ the wave function and n_λ the occupation number of the level λ (λ_F the Fermi level). MF model is a reasonable approximation if nucleon separation energy and momenta are not high (in nuclear ground state scale, $|\varepsilon| < 50\,\text{MeV}$ and $k < 300\,\text{MeV/c}$).

Fermi gas model (good for large nuclei)
$\mathcal{P}_{\rm MF}(\varepsilon,\boldsymbol{p}) = \theta(p_F-|\boldsymbol{p}|)\delta(\varepsilon - V - \boldsymbol{p}^2/2M)$.

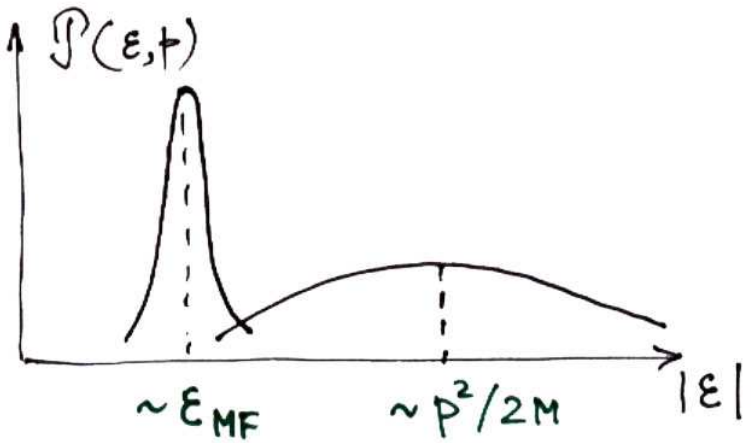
Nuclear spectral function

Nucleon short-range correlation effects

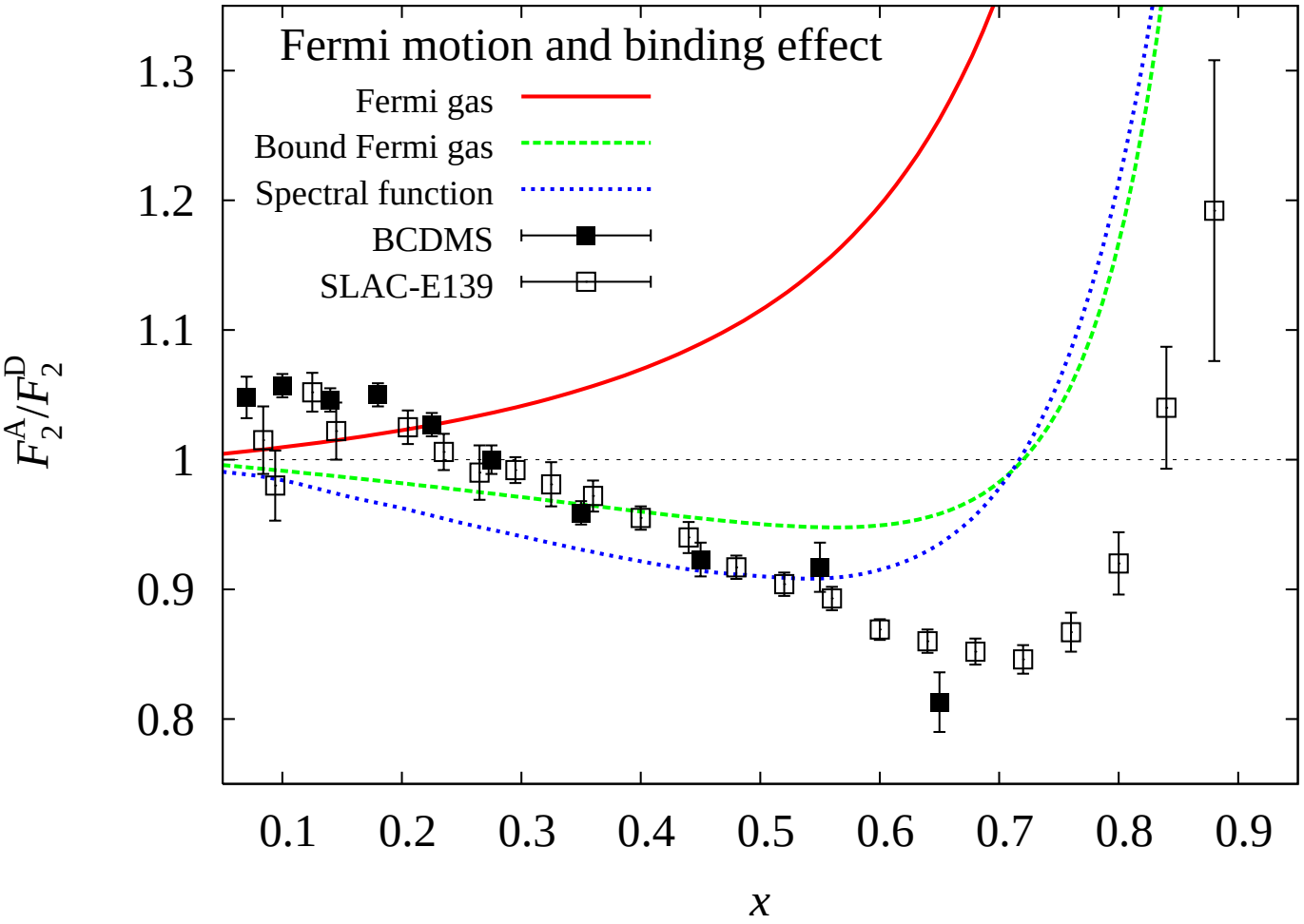
As the separation energy $|\varepsilon|$ becomes higher, the MF approximation becomes less accurate. High-energy and high-momentum component of nuclear spectrum can not be described in the MF model. These effects are driven by short-range NN correlations in nuclear ground state.

$$\mathcal{P}_{\rm cor}(\varepsilon,\boldsymbol{p}) \approx n_{\rm rel}(\boldsymbol{p}) \left\langle \delta\left(\varepsilon + \frac{(\boldsymbol{p}+\boldsymbol{p}_{A-2})^2}{2M} + E_{A-2} - E_A\right)\right\rangle_{A-2}$$

The full spectral function can be approximated by a sum of the MF and correlation parts $\mathcal{P} = \mathcal{P}_{\rm MF} + \mathcal{P}_{\rm cor}$.



EMC Ratio and Nuclear Spectral Function



Nucleon off-shell effect

Bound nucleons are off-mass-shell $k^2 = (M+\varepsilon)^2 - \boldsymbol{k}^2 < M^2$. In off-shell region nucleon structure functions depend on additional variable k^2.

A few model calculations exist for off-shell effect in DIS structure functions (Melnitchouk–Schreiber–Thomas, Gross–Liuti, Kulagin–Piller–Weise).

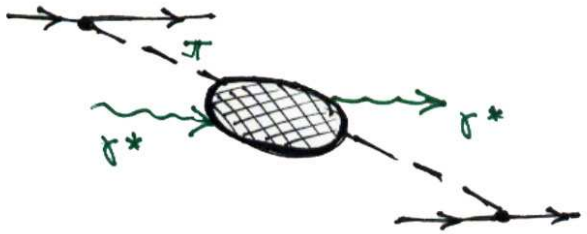
We follow phenomenological approach and extract off-shell dependence from data on the ratio of nuclear structure functions (S.K. & R.Petti, NPA765(2006)126). The virtuality parameter $v = (k^2 - M^2)/M^2$ (average virtuality $\langle v \rangle \sim -0.15$ for iron)

$$F_2^N(x,k^2) = F_2^N(x)\big(1 + v\,\delta f(x)\big)$$

Nuclear pion effect

Leptons can scatter on nuclear meson field which mediate interaction between bound nucleons. This process generate a pion correction to nuclear sea quark distribution (model calculations in the context of EMC effect by Llewellyn-Smith, Ericsson-Thomas, G.Miller,...)

$$\delta q^{\pi/A}(x,Q^2) = \int_x \frac{\mathrm{d}y}{y} f_{\pi/A}(y) q^{\pi}(\frac{x}{y},Q^2)$$



• Contribution from nuclear pions (mesons) is important to balance nuclear light-cone momentum $\langle y\rangle_\pi + \langle y\rangle_N = 1$.

• The nuclear pion distribution function is localized in a region of $y < p_F/M \sim 0.3$. For this reason the pion correction to nuclear (anti)quark distributions is localized at $x < 0.3$.

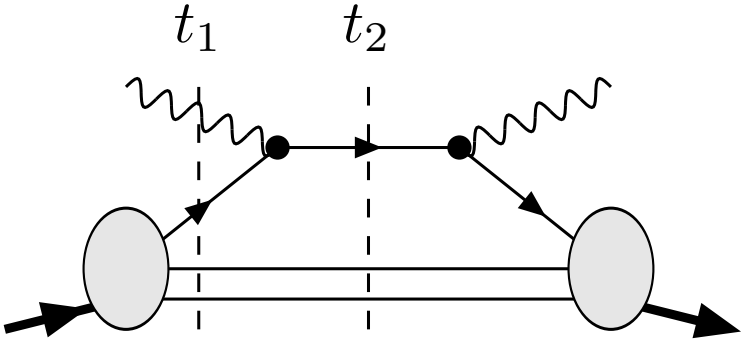
• The magnitude of the correction is driven by average number of "pions" $n_\pi = \int \mathrm{d}y\, f_{\pi/A}(y)$. By order of magnitude $n_\pi/A \sim 0.1$ for a heavy nucleus like ^{56}Fe.

• Nuclear pion correction effectively leads to enhancement of nuclear sea quark distribution and does not affect the valence quark distribution (for isoscalar nuclear target).

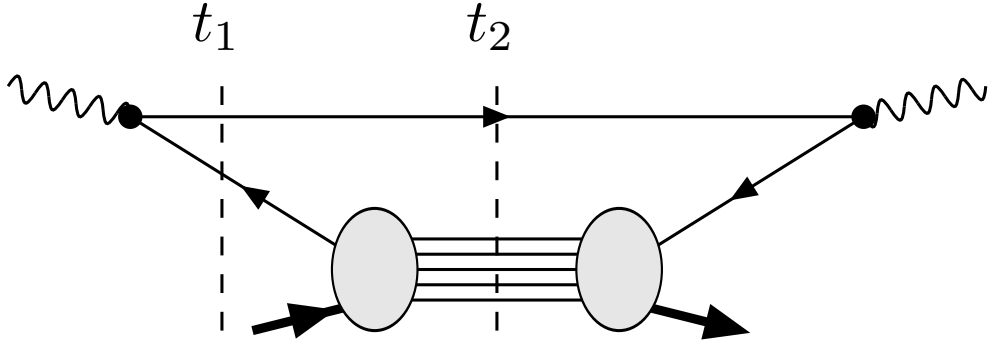
Coherent nuclear corrections

Two different mechanisms of DIS:

(I) QE scattering off bound quark. Characteristic interaction time $1/\langle\Delta E_I\rangle$ is finite and does not grow with $1/x$. This process dominates at large x and the structure functions are determined by quark wave functions.



(II) Intermediate virtual photon converts to quark-antiquark (hadronic) state $\gamma^* \to q\bar{q}$ then this state propagates in a target. Characteristic life time of the intermediate state grows with energy as $1/(Mx)$. This process dominates at small x and the structure functions are determined by quark scattering amplitudes.

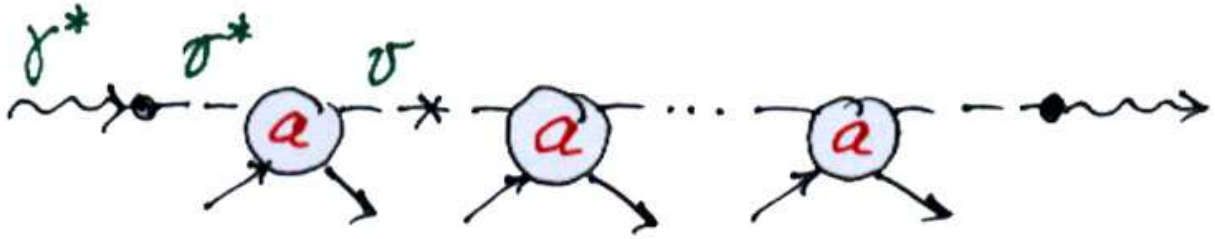


Nuclear effects arise due to interaction of intermediate states during propagation in matter. Relative correction to a single scattering term

$$\delta\mathcal{R} = \delta F_T^A / F_T^N \Rightarrow \delta\sigma_T^{\rm mult.sc.}/\sigma_T$$

Nuclear shadowing

Multiple scattering Glauber series



The series can be summed up in a compact form in optical approx. (suitable for large A)

$$\delta\mathcal{R} = \mathrm{Im}\left[ia^2\mathcal{C}_2^A(a)\right] / \mathrm{Im}\, a,$$

$$\mathcal{C}_2^A(a) = \int\limits_{z_1<z_2} \mathrm{d}^2\boldsymbol{b}\,\mathrm{d}z_1\mathrm{d}z_2\,\rho_A(\boldsymbol{b},z_1)\rho_A(\boldsymbol{b},z_2)\exp\left[i\int\limits_{z_1}^{z_2}\mathrm{d}z'\,(a\,\rho_A(\boldsymbol{b},z') - k_L)\right].$$

$a = \sigma(i+\alpha)/2$ is (effective) scattering amplitude in forward direction ($\alpha = \mathrm{Re}\, a/\mathrm{Im}\, a$), $k_L = Mx(1+m_v^2/Q^2)$ is longitudinal momentum transfer in the process $v^* \to v$ which accounts for finite life time of intermediate $q\bar{q}$ state. The presence of k_L suppresses nuclear mult. scat. effect as x increases.

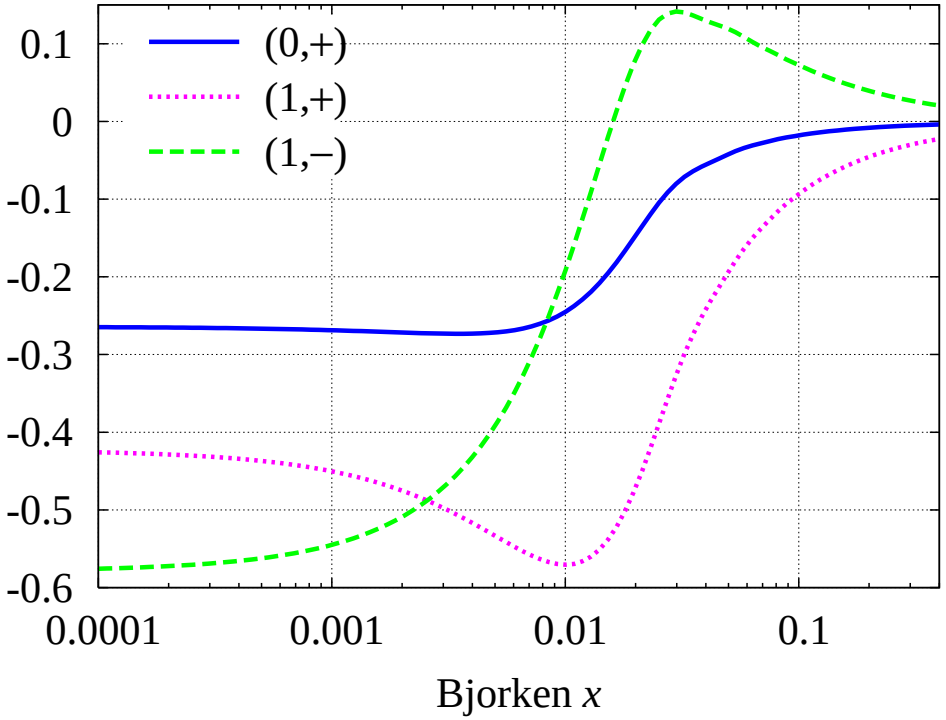
The amplitude a_h^I describes interaction of hadronic component of the virtual boson with a nucleon. This amplitude is characterized by helicity state h of the boson ($h = \pm 1$ for transverse polarization and $h = 0$ for longitudinal polariz.) and by isospin I (amplitude is different for proton and neutron).

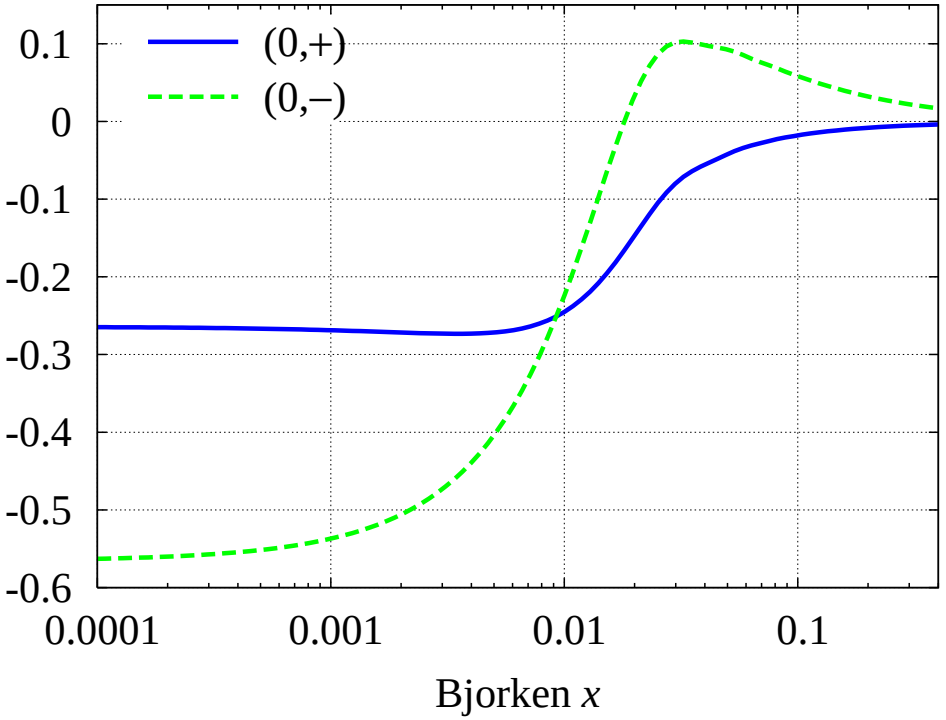
Average transverse amplitude $a_T^I = \frac{1}{2}(a_{+1}^I + a_{-1}^I)$
Helicity asymmetry $a_\Delta^I = \frac{1}{2}(a_{+1}^I - a_{-1}^I)$

Correspondence with structure functions:

$$a_T^0 \to F_T^{\nu+\bar\nu} \qquad a_\Delta^0 \to F_3^{\nu+\bar\nu}$$
$$a_T^1 \to F_T^{\nu-\bar\nu} \qquad a_\Delta^1 \to F_3^{\nu-\bar\nu}$$

Coherent nuclear effects are not universal but depend on structure functions.





The ratio $\delta\mathcal{R}^{(I,C)}$ calculated for different isospin and C-parity scattering states for ^{207}Pb at $Q^2 = 1\,\mathrm{GeV}^2$. The labels on the curves mark the values of the isospin I and C-parity, (I, C). From S.K. & R.Petti, hep-ph/0703033

Phenomenology of nuclear DIS

Motivation: The development of a quantitative model providing predictions of nuclear cross sections (structure functions) and corresponding uncertainties to be used in the analyses of present and future lepton scattering data from nuclear targets.

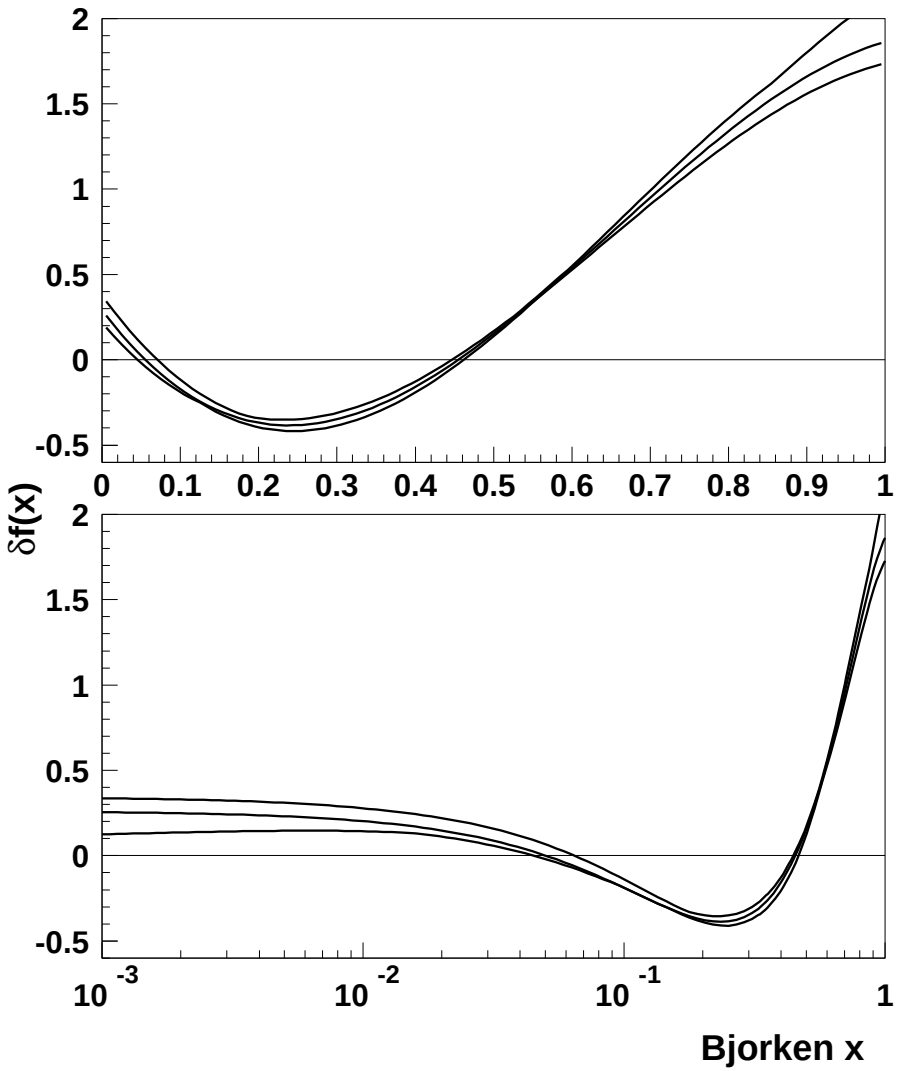
Approach: Take into account major nuclear mechanisms such as FMB, off-shell, nuclear pion and shadowing corrections. Parameterize off-shell correction function and effective scattering amplitude responsible for shadowing and extract these quantaties from data (for more detail see S.K. & R.Petti, NPA765(2006)126 and backup slides of this talk).

Results: Model is in a very good agreement with data for the entire kinematical region, $\chi^2/\mathrm{d.o.f} = 459/556$ for a 4 parameter fit.

Off-shell function

The function $\delta f(x)$ provides a measure of modification of quark distributions in bound nucleon.

$$\begin{aligned} \delta f(x) &= C_N(x-x_1)(x-x_0)(1+x_0-x) \\ C_N &= 8.1 \pm 0.3 \pm 0.5 \\ x_0 &= 0.448 \pm 0.005 \pm 0.007 \\ x_1 &= 0.05 \end{aligned}$$



- Parameters from the global fit (all nuclei) are consistent with independent fits to different subsets of nuclei
- The off-shell effect results in the enhancement of the structure function for $x_1 < x < x_0$ and depletion for $x < x_1$ and $x > x_0$.
- The parameters of $\delta f(x)$ suggest the increase in the radius of the bound nucleon valence region (in ^{56}Fe by $\sim 10\%$).

Adler Sum Rule

$$S_{\rm A} = \int_0^{M_A/M} {\rm d}x (F_2^{\bar\nu} - F_2^{\nu})/(2x) = 2I_z$$

- In parton model $S_{\rm A}$ is the difference between number of valence u and d in the target. $S_{\rm A}(p) = +1,\ S_{\rm A}(n) = -1$.

- ASR is independent of Q^2 and survives strong interaction effects because of CVC.

- Nonconservation of axial current is neglected in derivation of ASR. Therefore, ASR is good for sufficiently high Q^2 (ASR is violated by PCAC).

- For generic nucleus of Z protons and N neutrons $S_{\rm A}$ is given by neutron excess $S_{\rm A}/A = \beta = (Z-N)/A$.

Explicit calculation of different nuclear effects:
- FMB correction cancels out. So does the pion correction.
- Both off-shell (OS) and nuclear shadowing (NS) corrections are nonzero. The requirement of exact cancellation between these corrections fixes NS in isovector $(I = 1, C = -1)$ channel S.K. & R.Petti, hep-ph/0703033.

Gross–Llewellyn-Smith Sum Rule

$$S_{\rm GLS} = \frac{1}{2}\int_0^{M_A/M} {\rm d}x (F_3^{\bar\nu} + F_3^{\nu})$$

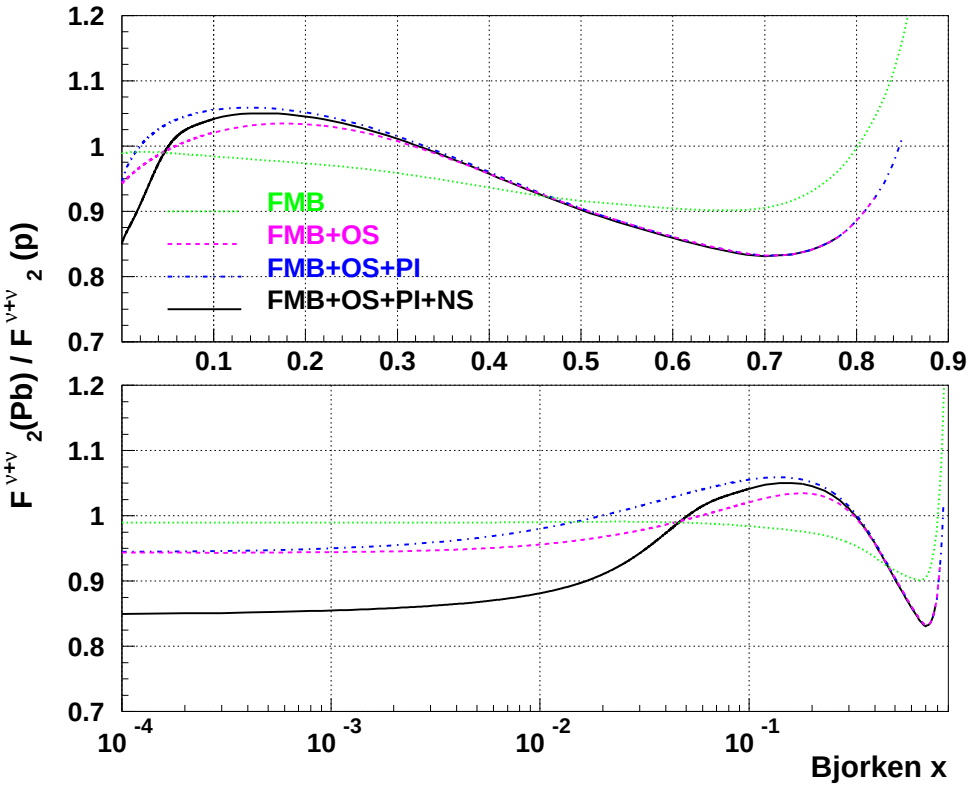
- In parton model $S_{\rm GLS} = 3$ is the number of valence u and d quarks in the target.

- In QCD the relation between $S_{\rm GLS}$ and the baryon number only holds at asymptotic Q^2. $S_{\rm GLS}$ is affected by QCD rad. cor. and HT effects

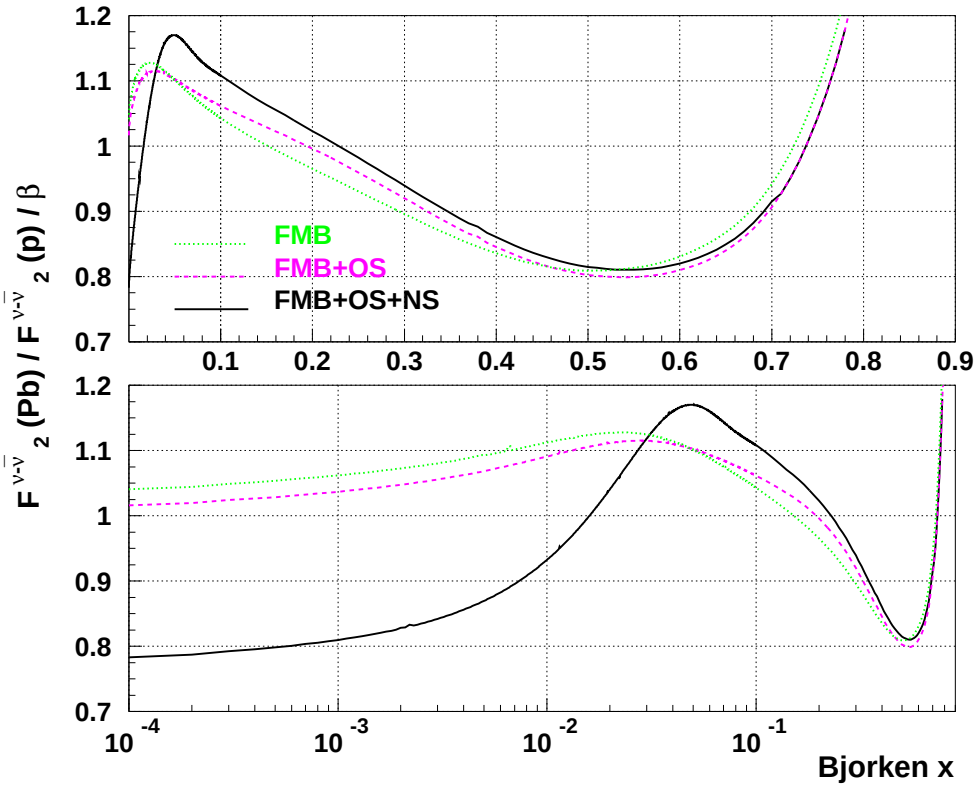
$$S_{\rm GLS} = 3(1 - \alpha_S/\pi - 3.25(\alpha_S/\pi)^2 + \cdots) + {\rm HT}_{\rm GLS}$$

Explicit calculation of different nuclear effects:
- FMB correction to $S_{\rm GLS}$ cancels out. Nuclear pion correction to $F_3^{\nu+\bar\nu}$ vanishes.
- Both off-shell (OS) and nuclear shadowing (NS) corrections are nonzero. The requirement of exact cancellation between these corrections helps to fix NS in isoscalar $(I = 0, C = -1)$ channel S.K. & R.Petti, hep-ph/0703033.

Dependence of Nuclear Effects on C parity

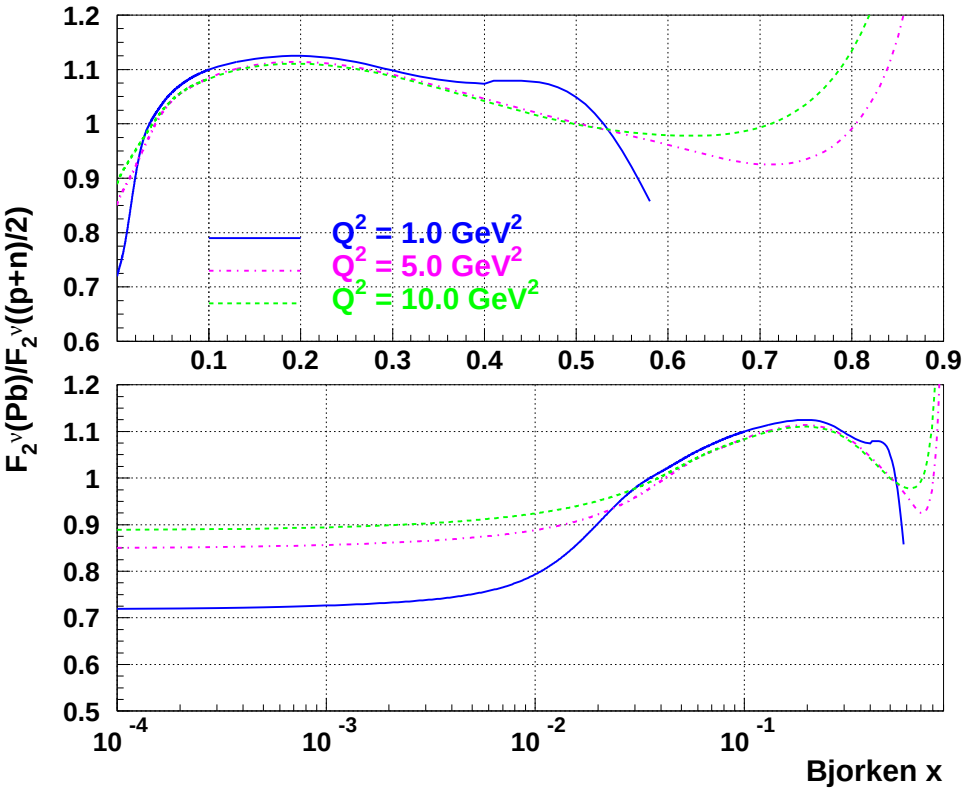


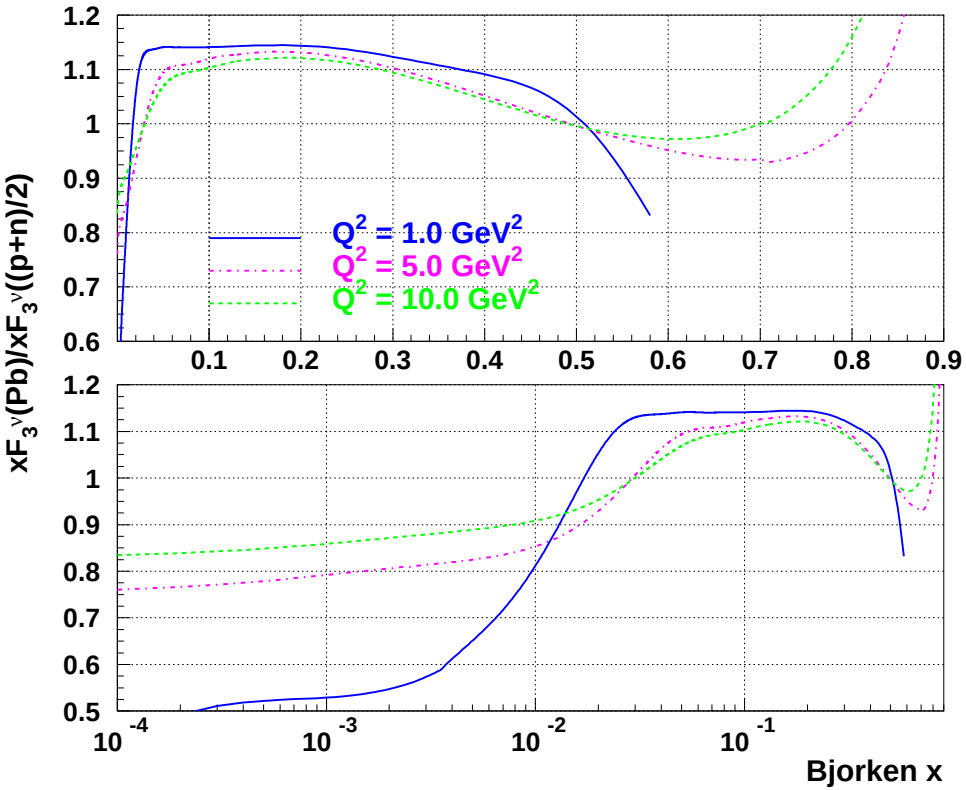
The ratio $\frac{1}{A}F_2^{(\nu+\bar\nu)A}/F_2^{(\nu+\bar\nu)p}$ calculated for ^{207}Pb at $Q^2 = 5\ \mathrm{GeV}^2$. The labels on the curves correspond to effects due to Fermi motion and nuclear binding (FMB), off-shell correction (OS), nuclear pion excess (PI) and coherent nuclear processes (NS).



The ratio $\frac{1}{A}F_2^{(\nu-\bar\nu)A}/(\beta F_2^{(\nu-\bar\nu)p})$ calculated for ^{207}Pb at $Q^2 = 5\,\mathrm{GeV}^2$. The labels on the curves correspond to effects due to Fermi motion and nuclear binding (FMB), off-shell correction (OS) and coherent nuclear processes (NS).

Combined Nuclear Effects for F_2^ν and F_3^ν





The ratios of CC neutrino structure functions for ^{207}Pb normalized to one nucleon and those of the isoscalar nucleon (p+n)/2 (left panel for F_2 and right panel for xF_3).

Summary

A number of nuclear effects common for CL and neutrino DIS was discussed.

In general nuclear effects are not universal even if the underlying physics seems to be similar for CL and neutrino scattering.

Although nuclear effects measured in CL DIS cannot be taken over to neutrino interactions, the combined studies of CL and neutrino data in terms of a unified approach are very useful in fixing the description of nuclear effects in neutrino scattering. This is the way on which theoreticians can help to reduce systematic uncertainties in neutrino experiments and give a correct interpretation of data.

Backup slides

Isospin effects in nuclear convolution

Separation of isospin 0 anf 1 contributions in nuclear convolution:

$$\mathcal{P}_p F^p + \mathcal{P}_n F^n = \tfrac{1}{2}\mathcal{P}_{p+n} F^{p+n} + \tfrac{1}{2}\mathcal{P}_{p-n} F^{p-n},$$
$$\mathcal{P}_{p\pm n} = \mathcal{P}_p \pm \mathcal{P}_n$$

The reduced isoscalar and isovector spectral functions (normalized to unity)

$$\mathcal{P}_{p+n} = A\,\mathcal{P}_0,$$
$$\mathcal{P}_{p-n} = (Z-N)\mathcal{P}_1$$

$\mathcal{P}_0$ involves the averaging over isoscalar intermediate states and $\mathcal{P}_1$ is isovector distribution in a nucleus.

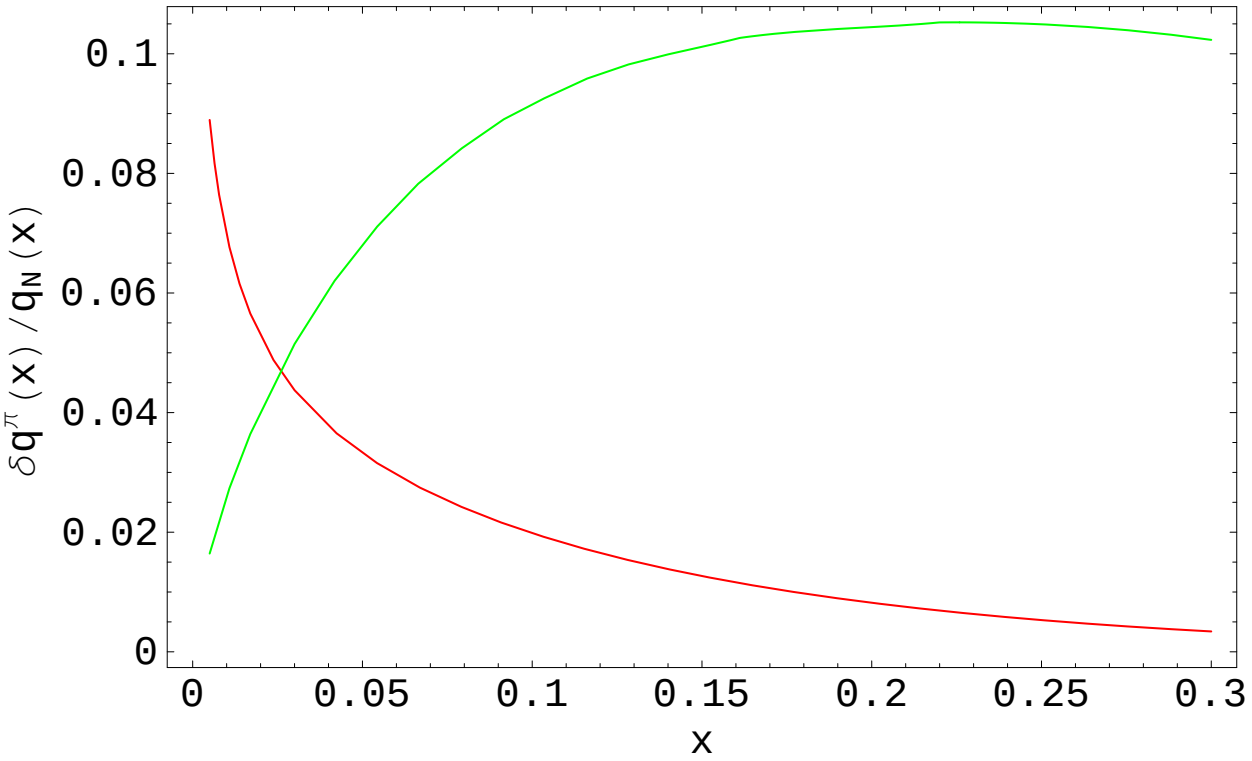
Isovector nuclear spectral function

If the differences in the proton and neutron energies (wave functions) can be neglected (neglect Coulomb effect) then $\mathcal{P}_1$ is given by the difference in the level occupation numbers $n_\lambda^{\rm proton} - n_\lambda^{\rm neutron}$. Because of Pauli principle $\mathcal{P}_1$ in complex nuclei is determined by the contribution from the Fermi level

$$\mathcal{P}_1 = |\phi_F(\boldsymbol{p})|^2 \delta(\varepsilon - \varepsilon_F),$$

ε_F and ϕ_F are the energy and the wave function of the Fermi level. In Fermi gas model $|\phi_F(\boldsymbol{p})|^2 = \delta(p - p_F)/(4\pi p_F^2)$

Nuclear Pion Correction



The ratio of the pion correction to nuclear antiquark distribution and the nucleon valence quark distribution (red) and nucleon antiquark distribution (green) calculated for ^{56}Fe and $Q^2 = 20\,\mathrm{GeV}^2$.

Analysis

We use the data from electron and muon DIS in the form of ratios $\mathcal{R}_2(A/B) = F_2^A/F_2^B$ for a variaty of targets. The data are available for A/D and $A/^{12}\mathrm{C}$ ratios.

We calculate nuclear structure functions taking into account major nuclear mechanisms such as FMB, off-shell, nuclear pion and shadowing corrections and perform a fit to minimize $\chi^2 = \sum_{\mathrm{data}}(\mathcal{R}_2^{exp} - \mathcal{R}_2^{th})^2/\sigma^2(\mathcal{R}_2^{exp})$ with σ the experimental uncertainty. In the fit we use data with $Q^2 > 1\,\mathrm{GeV}^2$ (overall about 560 points). Then we validate the predictions for $Q^2 < 1\,\mathrm{GeV}^2$.

We use the parametrization $\delta f_2(x) = C_N(x - x_1)(x - x_0)(h - x)$ for the off-shell function. For the effective scattering amplitude of hadronic component of γ^* off the nucleon we use the following model

$$\bar{a}_T = \bar{\sigma}_T(i + \alpha)/2, \qquad \bar{\sigma}_T = \sigma_1 + \frac{\sigma_0 - \sigma_1}{1 + Q^2/Q_0^2}$$

Preliminary trials:

Initialy the parameters $\sigma_1,\ \sigma_0,\ \alpha_T,\ Q_0^2$ and $x_1,\ x_0,\ h,\ C_N$ are free and we study the correlations between them.

$*$ The parameter h turned out fully correlated with x_0, $h = 1 + x_0$ fixed in the final fit.

$*$ Best fit gives $\sigma_1 \approx 0$. The correlations between σ_1 and off-shell parameters are negligible. We fix $\sigma_1 = 0$.

$*$ The parameter σ_0 is strongly correlated $\sigma_0 \leftrightarrow Q_0^2$, $\sigma_0 \leftrightarrow \alpha_T$. Best fit gives $\alpha_T = -0.179 \pm 0.038$ and $\Delta\chi^2 \sim 29$ compared to the fit with $\alpha_T = 0$. If we fix $\sigma_0 = 27\,\text{mb}$ (average cross section in the VMD model) then $\alpha_T = -0.182 \pm 0.037$. This is in good agreement with results on the analysis of ρ^0 meson production in VMD. In final fit we fix $\sigma_0 = 27\,\text{mb}$ and $\alpha = -0.2$.

The ajustable parameters of the model are $x_0,\ x_1,\ C_N$ and Q_0^2.

Additional constraint of the fit is the normalization of nuclear valence quark number. This allows us to constrain x_1. For this purpose we use an iterative algorithm:

1. Fit with fixed x_1 without normalization constraint.
2. Calculate $\delta N_{\rm val} = \delta N_{\rm val}^{\rm off-shell} + \delta N_{\rm val}^{\rm shadowing}$ as a function of Q^2 at $Q^2 > 5\,{\rm GeV}^2$
3. Change x_1 and go to 1. until $|\delta N_{\rm val}|$ is minimized provided that χ^2 is still "acceptable".

Results

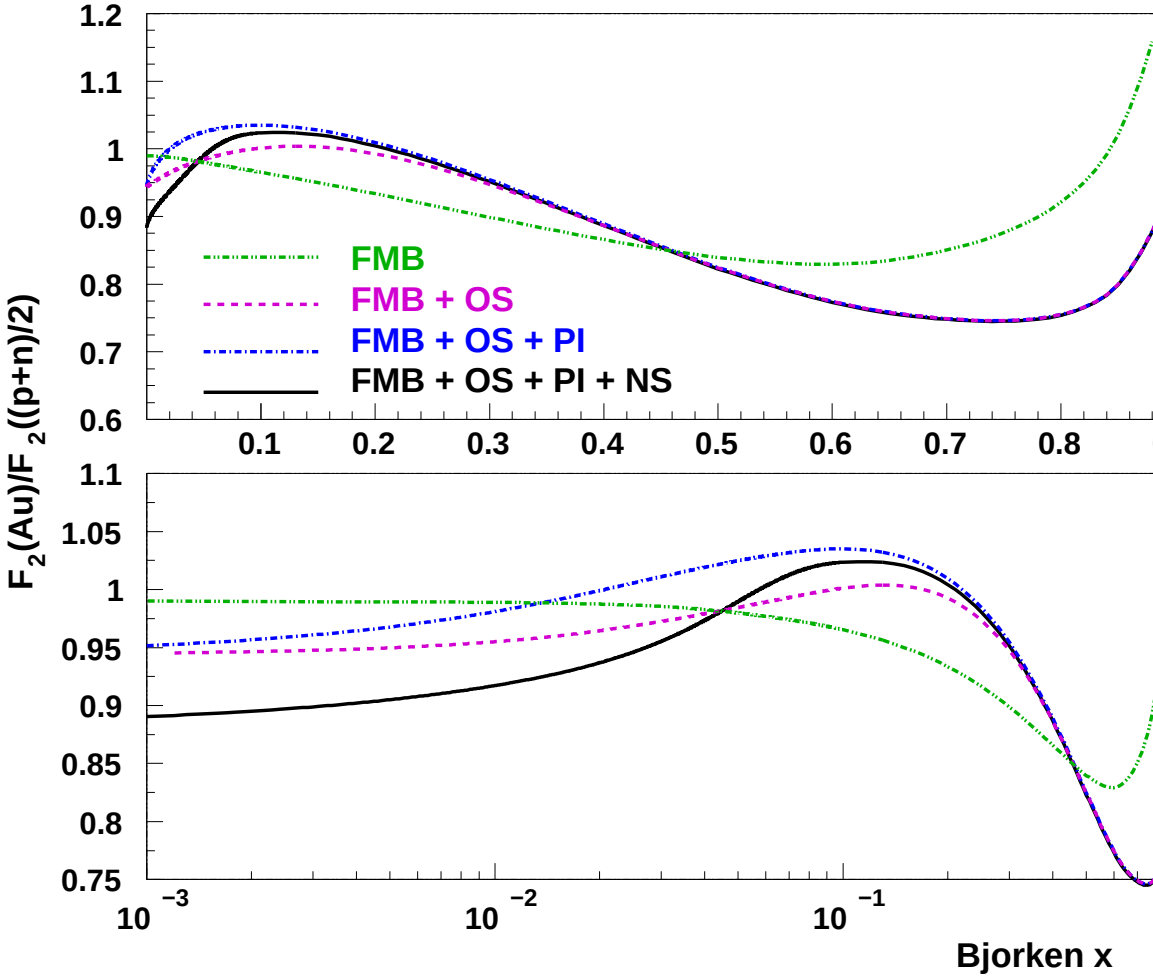
- Very good agreement with data for the entire kinematical region

$$\chi^2/\text{d.o.f} = 459/556$$

- Parameters from the global fit (all nuclei) are consistent with independent fits to different subsets of nuclei

$$C_N = 8.1 \pm 0.3$$
$$x_0 = 0.448 \pm 0.005$$
$$Q_0^2 = 1.43 \pm 0.06 \ \text{GeV}^2$$

Systematic/theoretical uncertainties:
$\delta C_N = 0.5$, $\delta x_0 = 0.007$, $\delta Q_0^2 = 0.2 \ \text{GeV}^2$

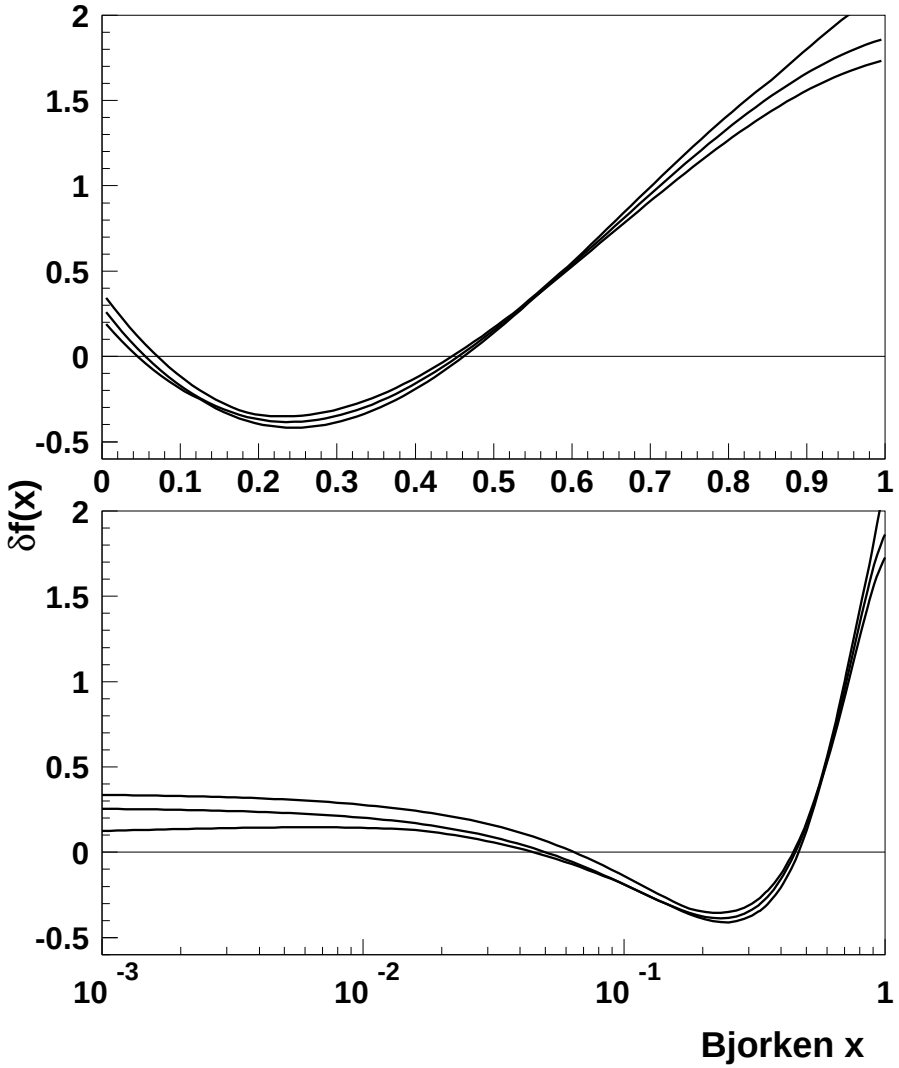


Off-shell function

- The function $\delta f(x)$ provides a measure of modification of quark distributions in bound nucleon.

- The off-shell effect results in the enhancement of the structure function for $x_1 < x < x_0$ and depletion for $x < x_1$ and $x > x_0$.

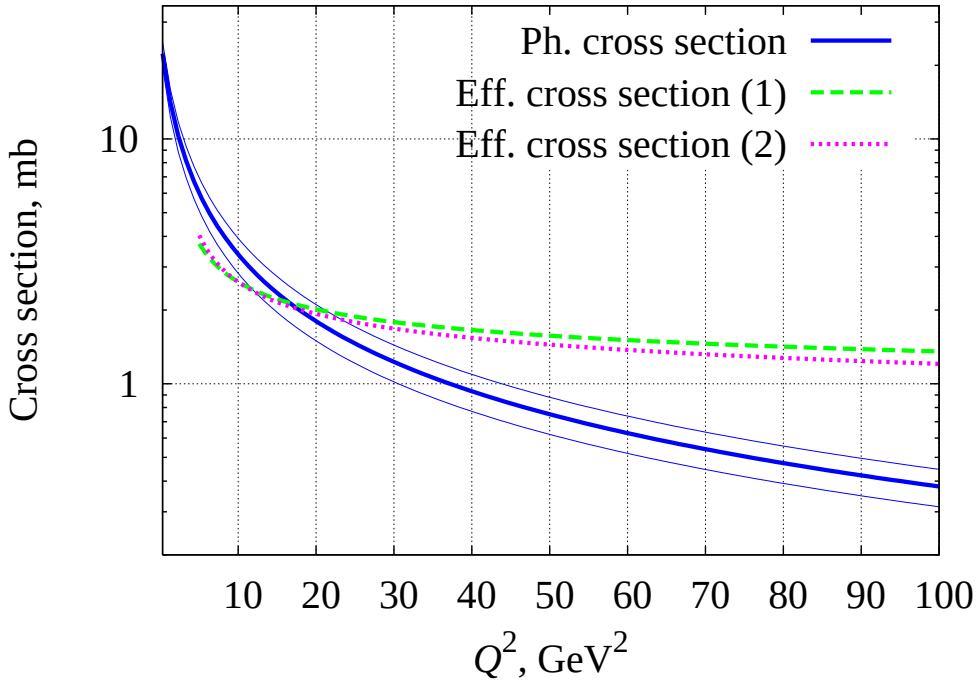
- The parameters of $\delta f(x)$ suggest the increase in the radius of the bound nucleon valence region (in Fe by $\sim 10\%$).



Effective cross section

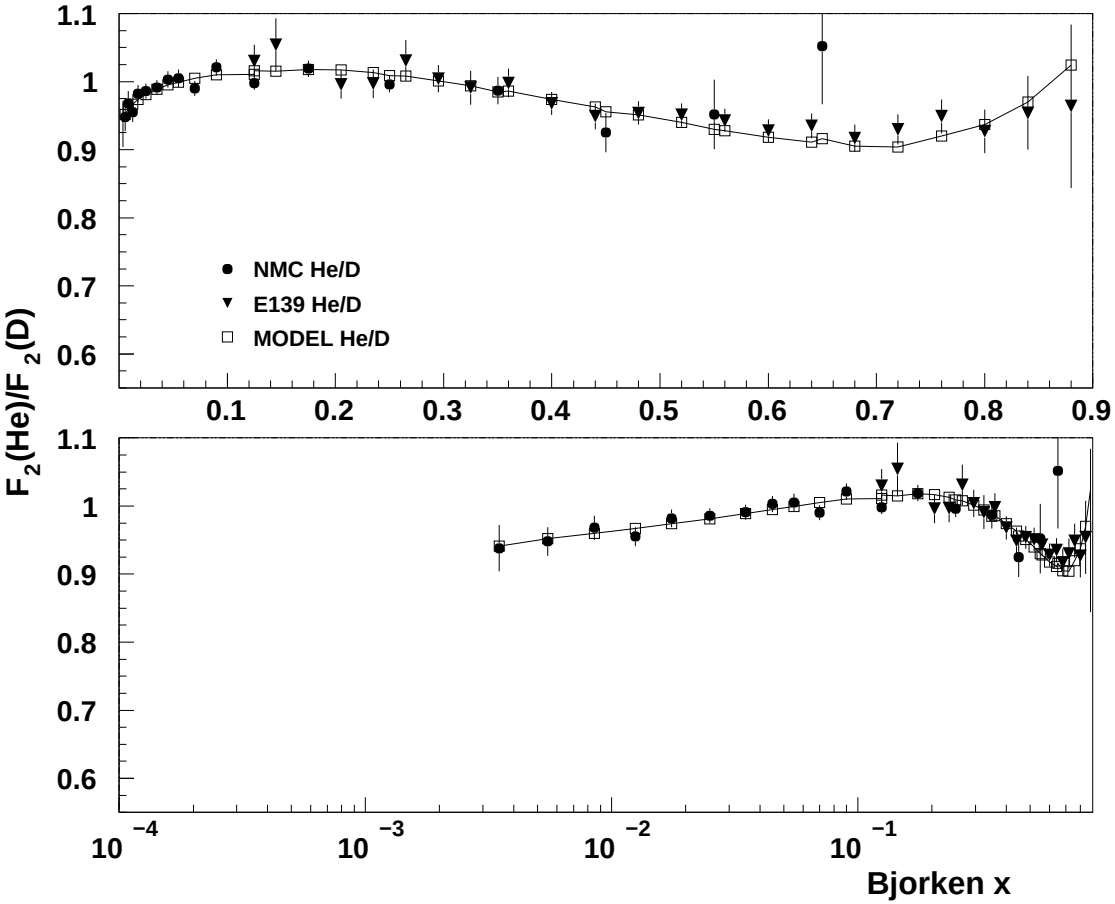
- The monopole form $\bar{\sigma} = \sigma_0/(1 + Q^2/Q_0^2)$ provides a good fit to existing data on nuclear shadowing for $Q^2 < 20\ \mathrm{GeV}^2$ (ratio $F_2(\mathrm{Sn})/F_2(\mathrm{C})$ from NMC).

- This does not necessarily mean that $\bar{\sigma}$ vanish as $1/Q^2$ at very high momentum transfer since data are limited to $Q^2 < 20\ \mathrm{GeV}^2$. Effective cross section at high Q^2 can be calculated from the normalization condition of the valence quark distribution $\delta N_{\mathrm{val}}^{\mathrm{off-shell}} + \delta N_{\mathrm{val}}^{\mathrm{shadowing}} = 0$.

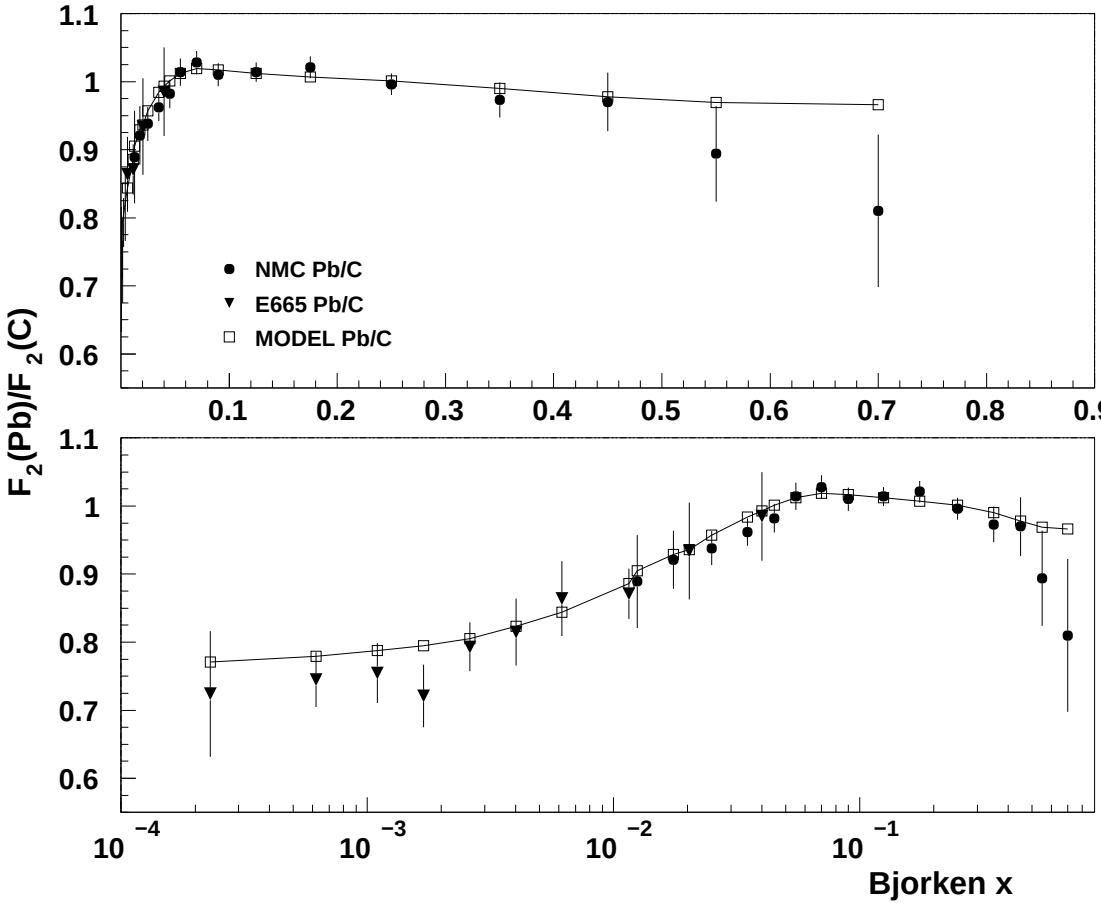


Ratios $\mathcal{R}_2(x, A/B) = F_2^A/F_2^B$

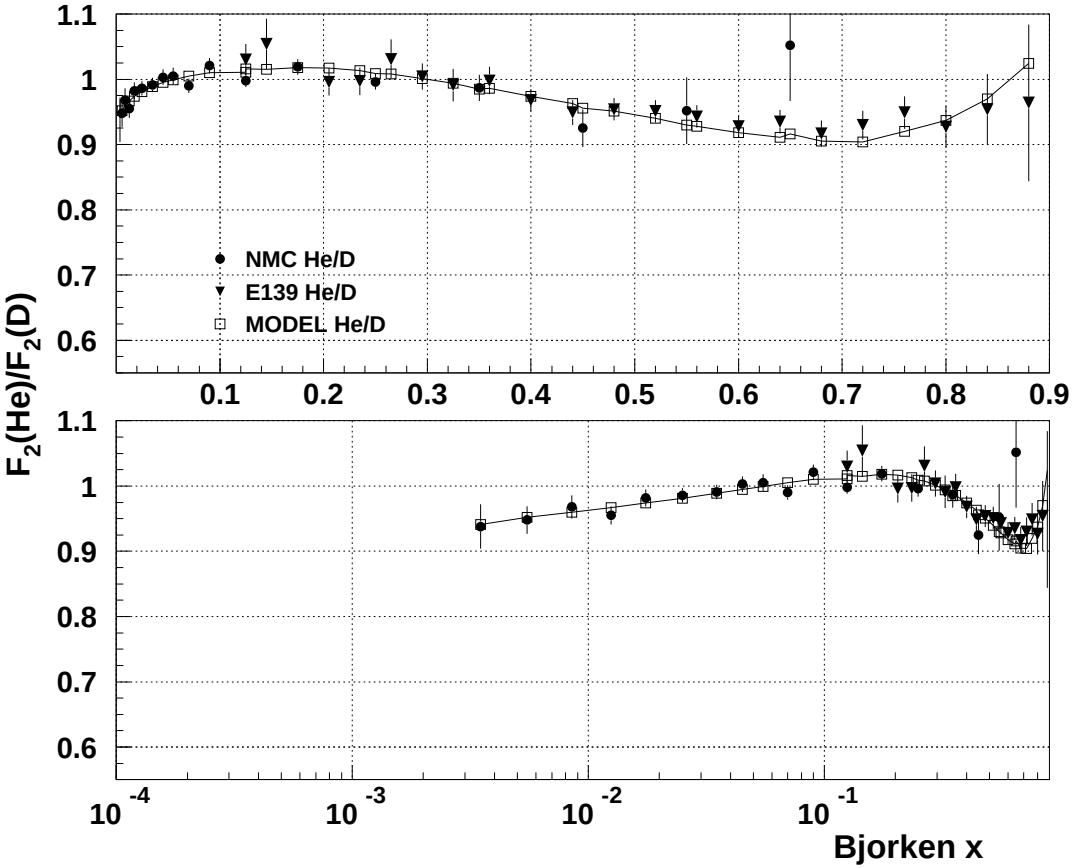
^{4}He/D



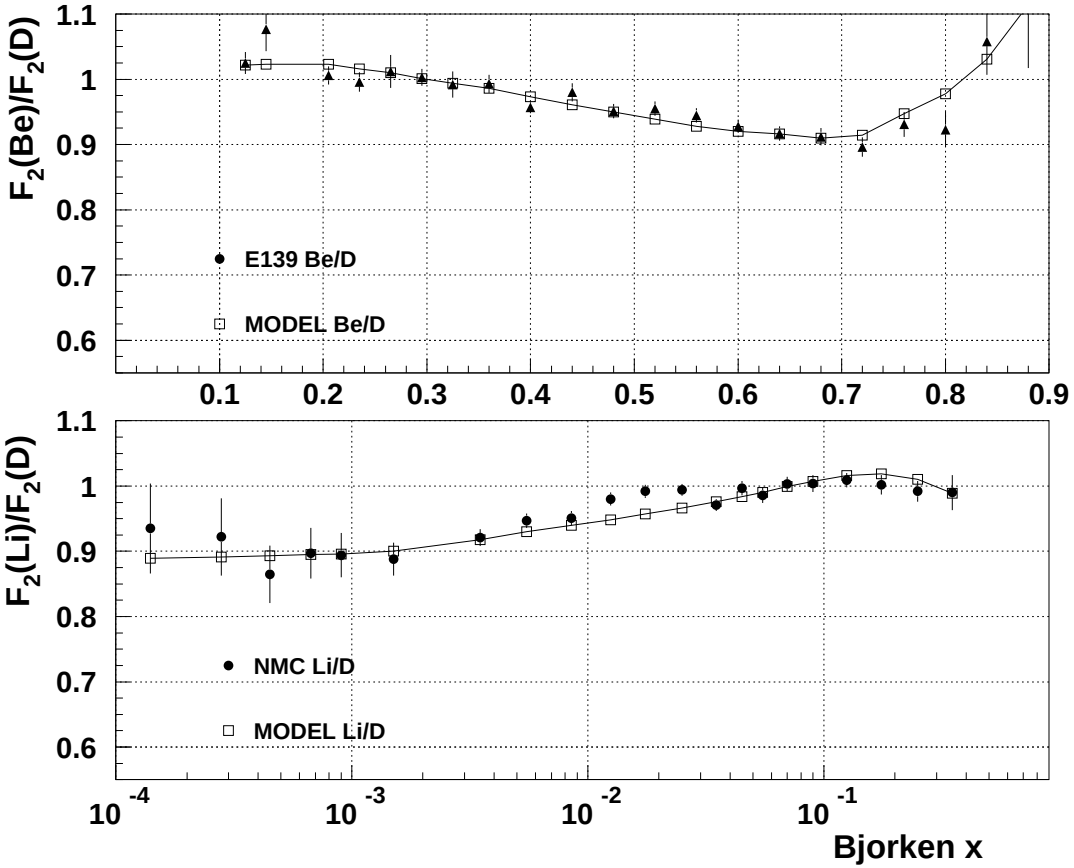
^{208}Pb/^{12}C



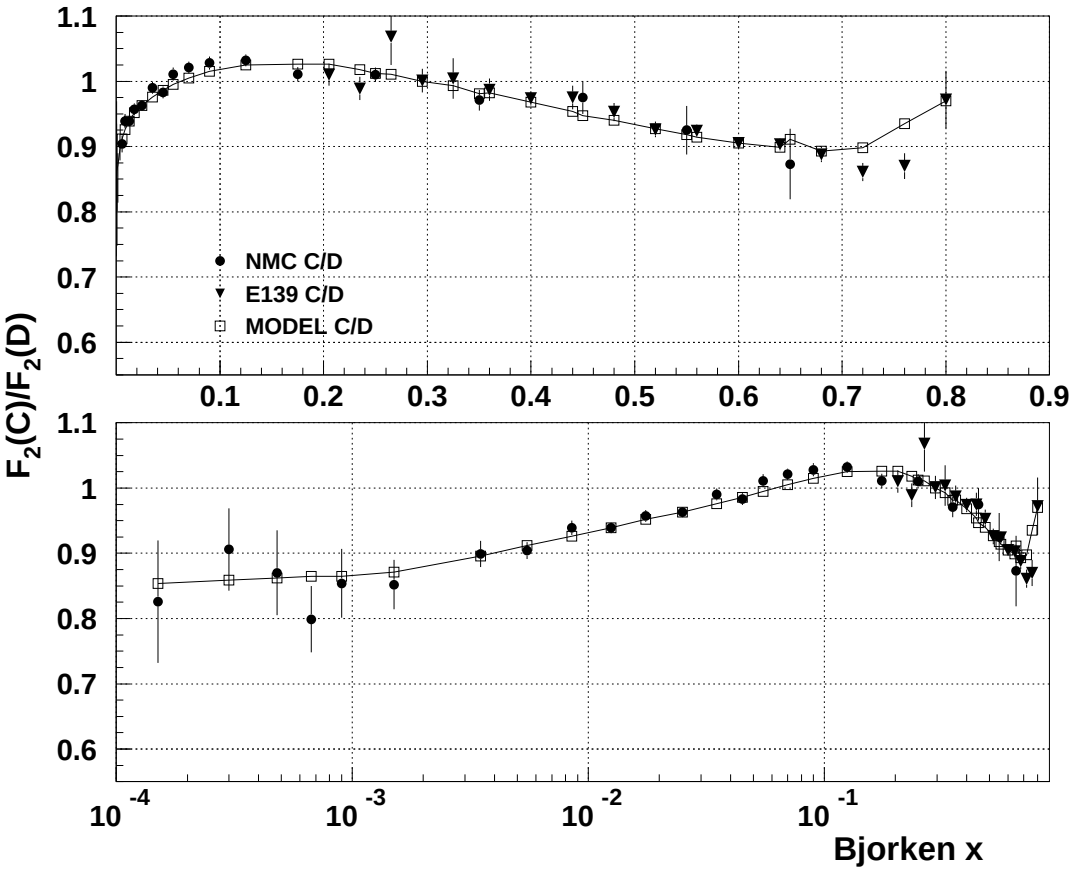
^{4}He/D



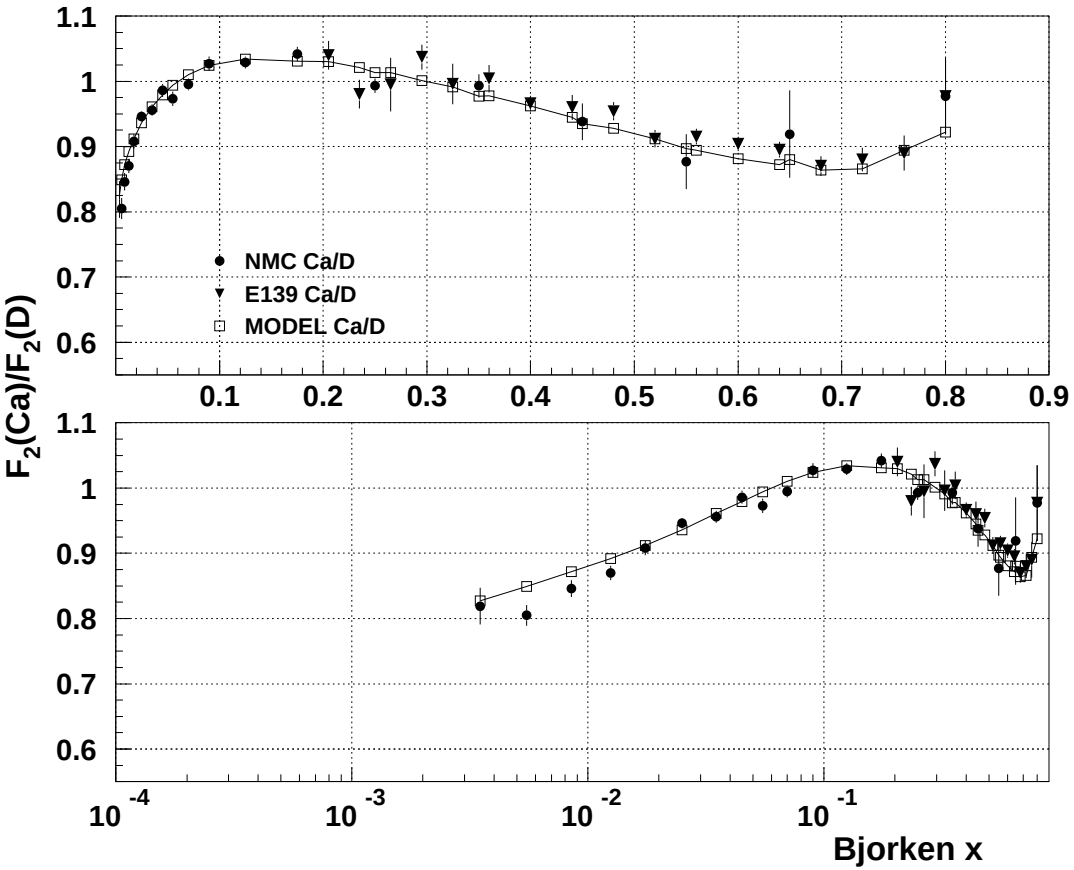
^{7}Li/D & ^{9}Be/D



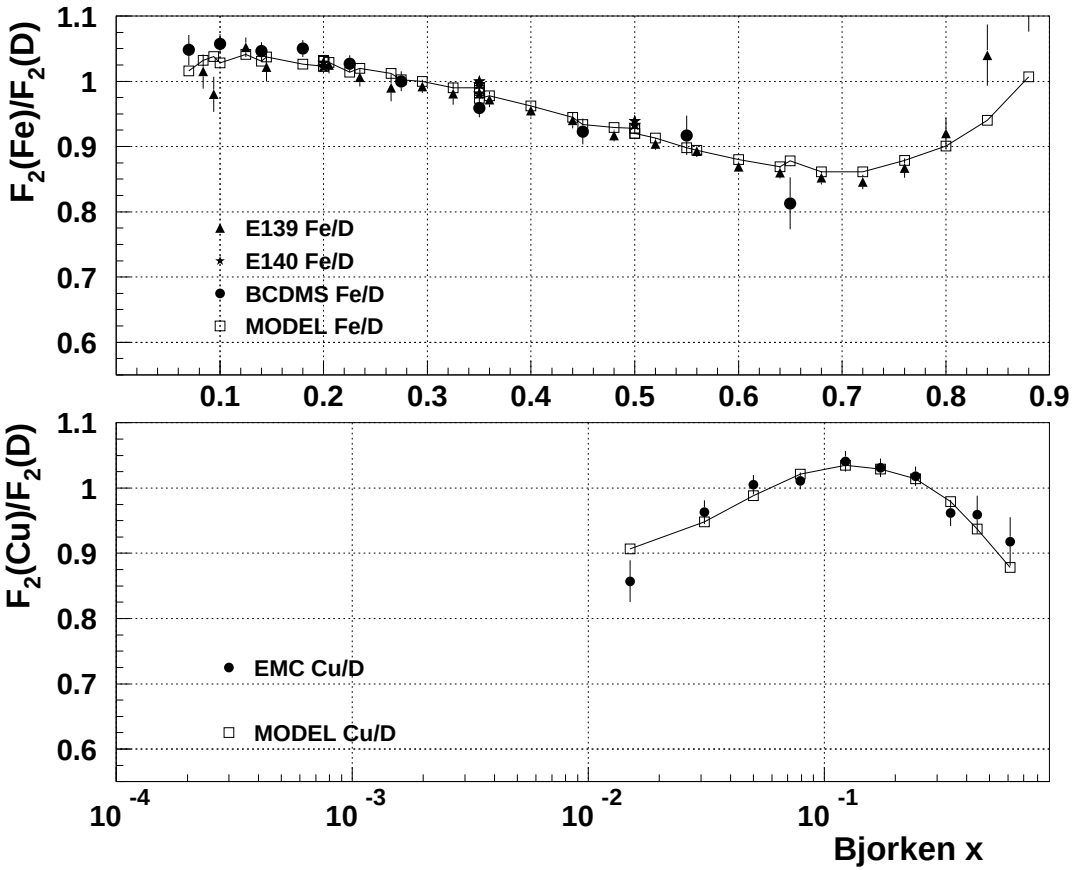
$^{12}C/D$


NMC C/D
E139 C/D
MODEL C/D
F2(C)/F2(D)
Bjorken x

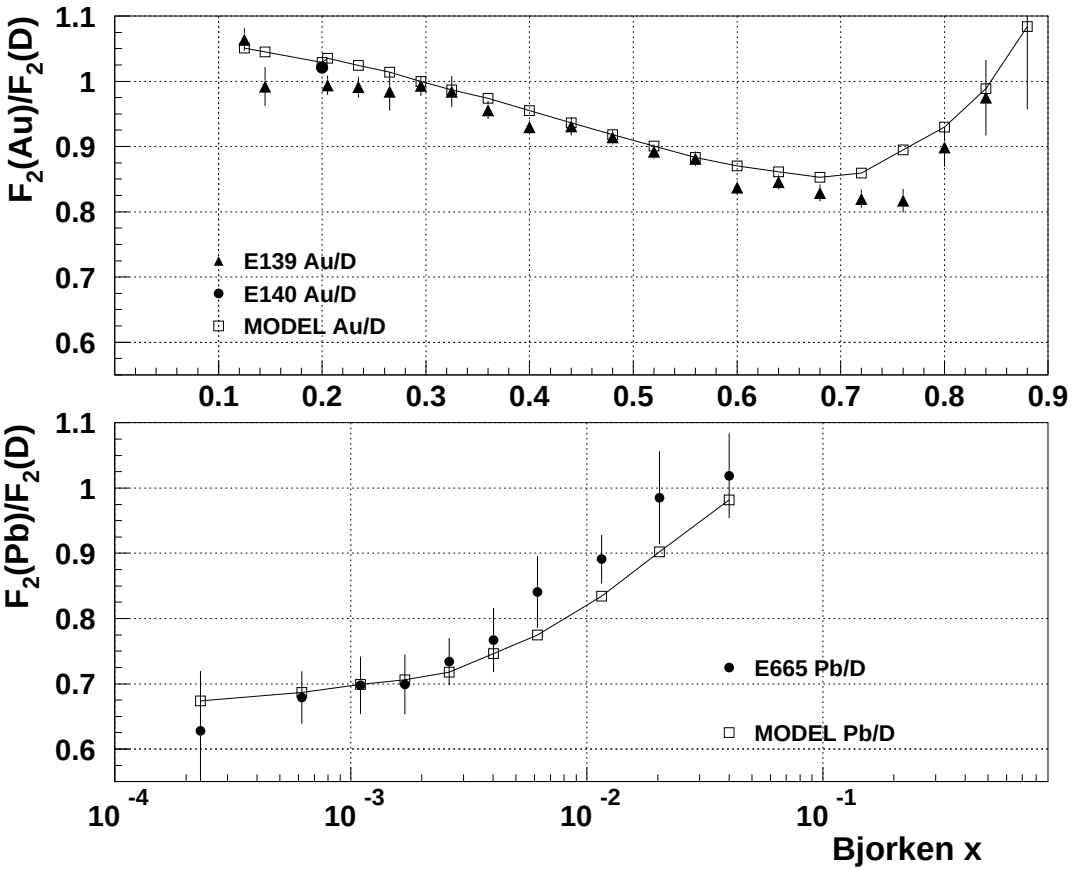
$^{40}Ca/D$


NMC Ca/D
E139 Ca/D
MODEL Ca/D
F2(Ca)/F2(D)
Bjorken x

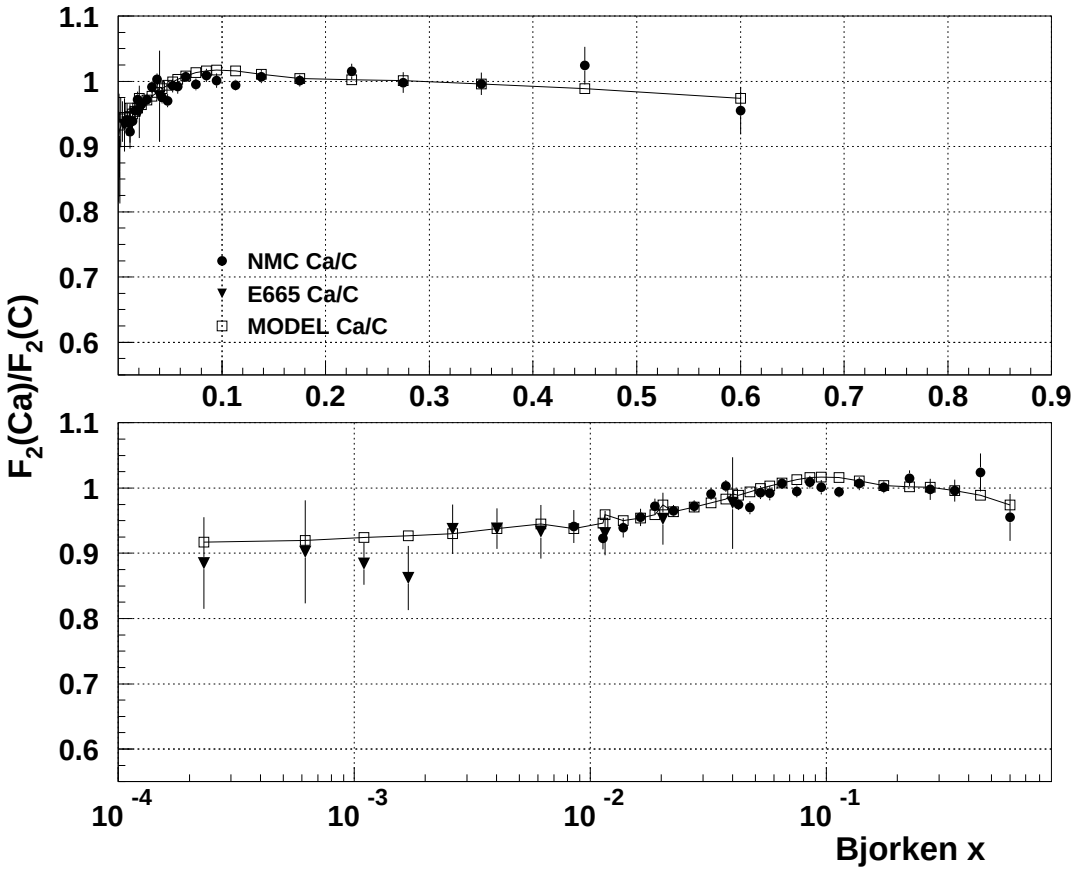
^{56}Fe/D & ^{63}Cu/D



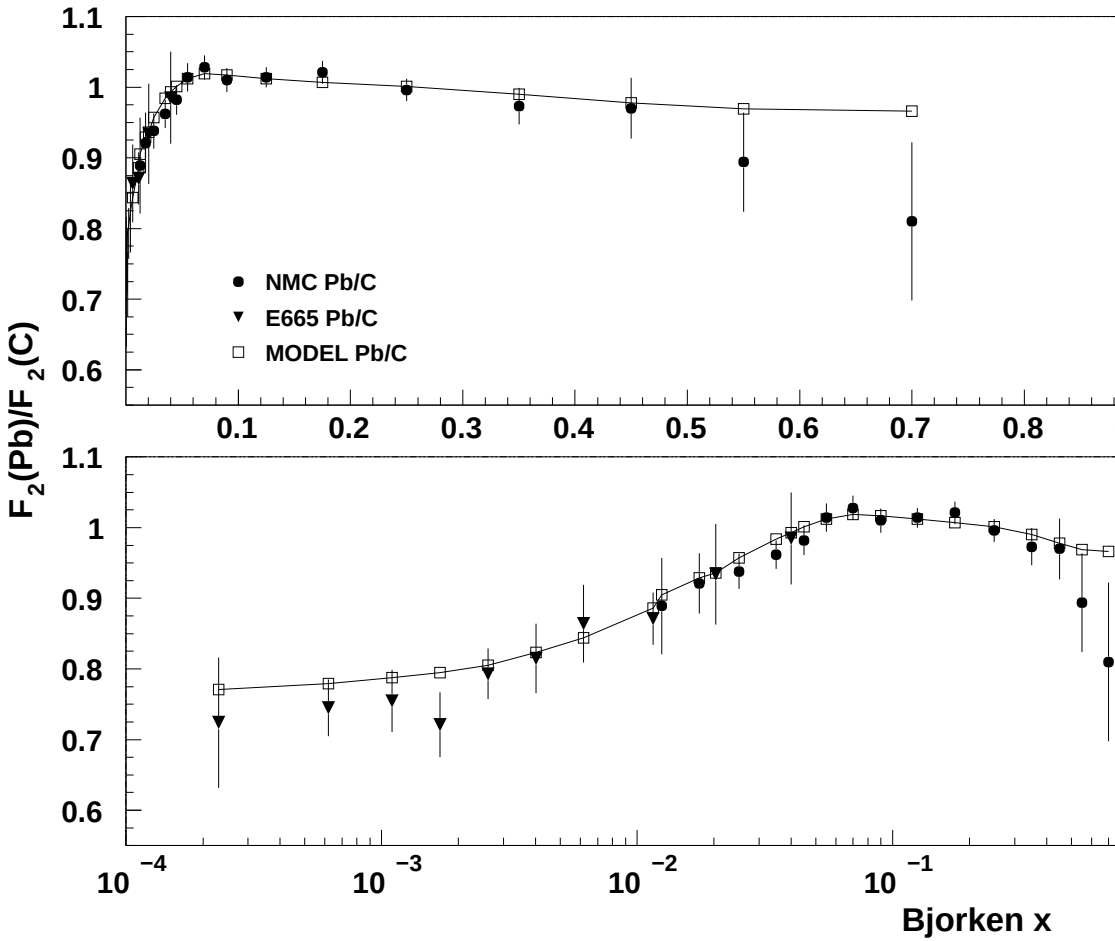
^{197}Au/D & ^{207}Pb/D



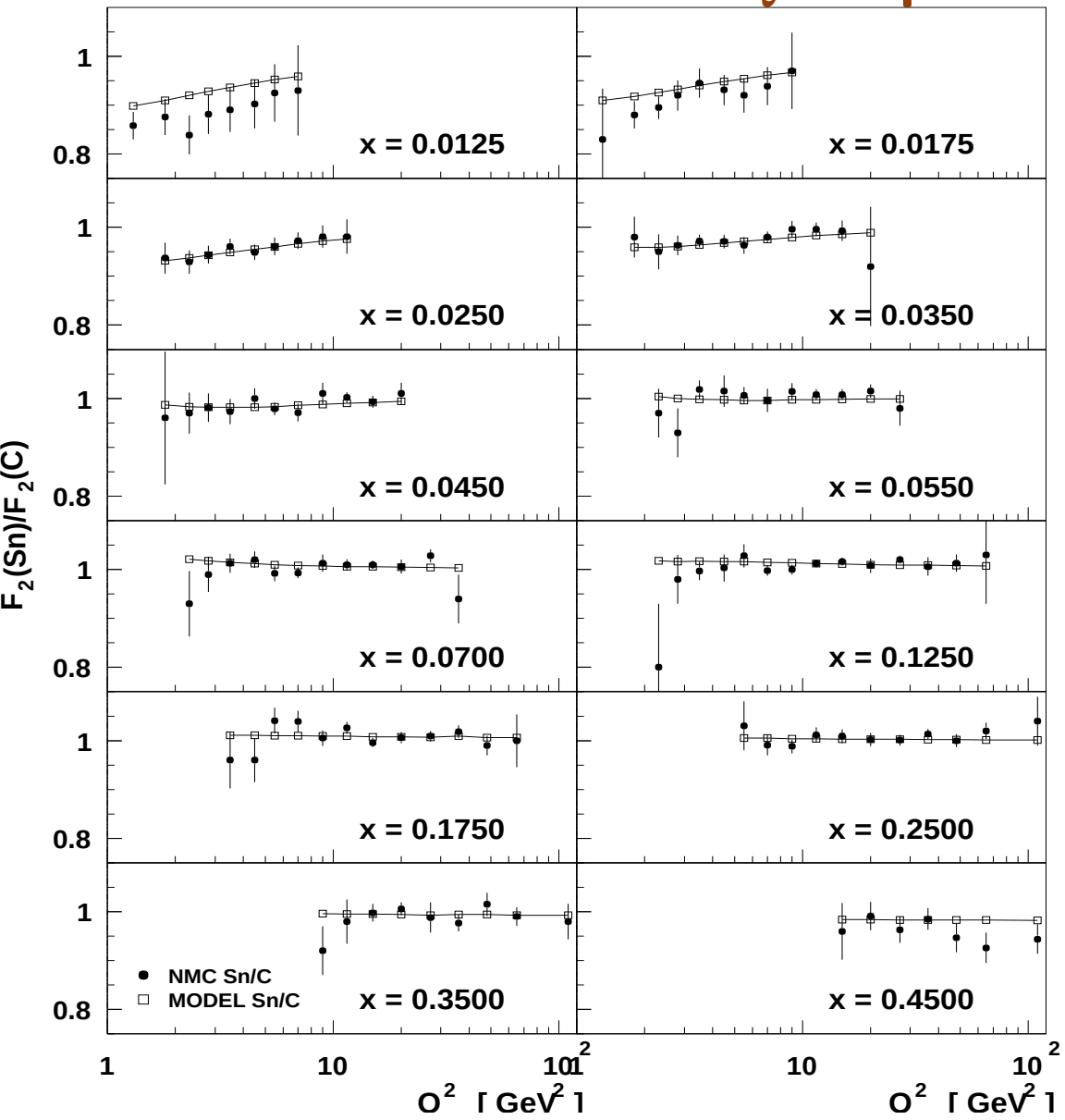
$^{40}Ca/^{12}C$

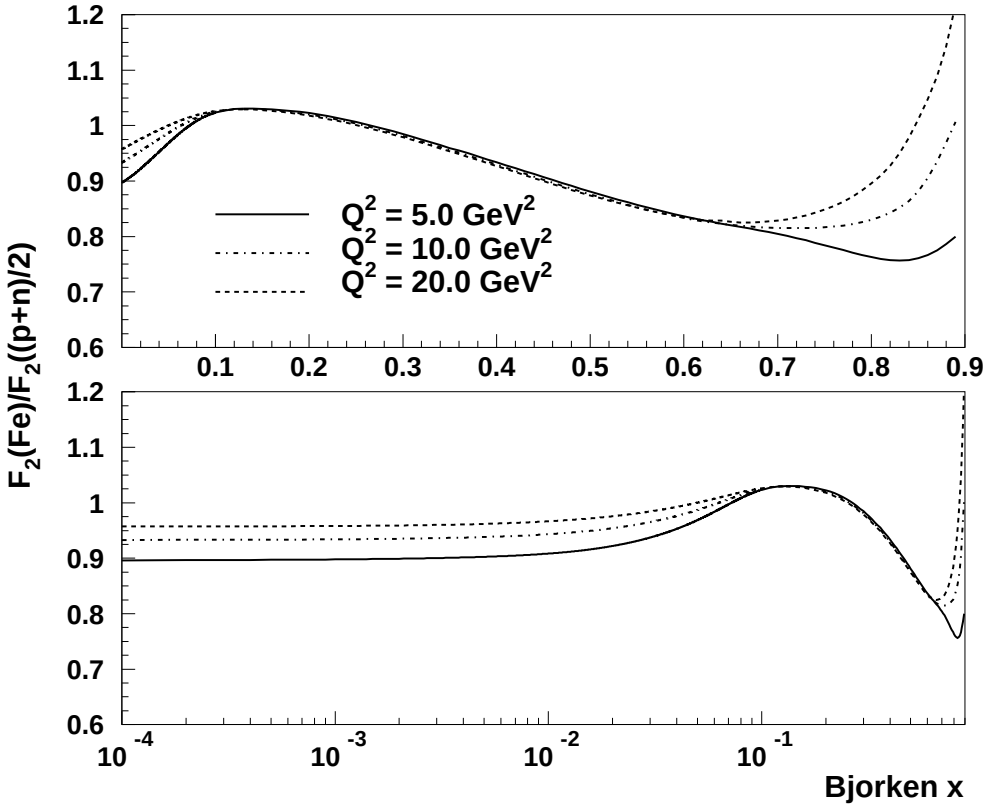


$^{207}Pb/^{12}C$



Q^2 dependence of $\mathcal{R}_2$





Significant Q^2 dependence observed at $x < 0.05$ (the Q^2 dependence of shadowing effect) and for $x > 0.7$ (the Q^2 dependence of target mass correction)